

PART 6-Simplify Rational Expressions

1. $\frac{x^3-2x^2+x-2}{x^4-x^2-2}$

2. $\frac{t^4-1}{t^3+t^2+t+1}$

SOLUTIONS

PART 7- Multiply and Divide Rational Expressions

1. $\frac{\frac{x^2-y^2}{x+y}}{x^4-y^4}$

2. $\frac{3x^2-6x}{x^2-6x+9} \times \frac{x^2-x-6}{x^2-4}$

PART 8-Adding and Subtracting Rational Expressions

1. $\frac{1}{t^2+t} + \frac{1}{t^2-t}$

2. $\frac{x}{x-a} - \frac{x^2+a^2}{x^2-a^2} + \frac{a}{x+a}$

PART 9- Complex Fractions

1. $\frac{\frac{2}{y+2} - 1}{\frac{1}{y+2} + 1}$

2. $\frac{k + \frac{1}{k-2}}{\frac{k^2}{k-2} + 1}$

PART 10-Rational Equations

1. $\frac{z+3}{3-z} + \frac{11z+3}{z^2-9} = \frac{1-5z}{z+3}$

2. $\frac{(y+1)^2}{(y-3)^2} = 1$

PART 17-Radical Equations

1. $\sqrt{2x-2} - \sqrt{x+6} = 1$

2. $3x = 2 + x\sqrt{5}$

Part 6

$$\begin{aligned} \textcircled{1} \frac{(x^3 - 2x^2) + (x - 2)}{x^4 - x^2 - 2} &= \frac{x^2(x-2) + 1(x-2)}{(x^2-2)(x+1)} \\ &= \frac{\cancel{(x^2-2)}(x-2)}{(x^2-2)\cancel{(x+1)}} \\ &= \boxed{\frac{x-2}{x^2-2}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \frac{t^4 - 1}{(t^3 + t^2) + (t + 1)} &= \frac{(t^2+1)(t^2-1)}{t^2(t+1) + 1(t+1)} \\ &= \frac{\cancel{(t^2+1)}\cancel{(t+1)}(t-1)}{\cancel{(t^2+1)}\cancel{(t+1)}} \\ &= \boxed{t-1} \end{aligned}$$

Part 7

$$\begin{aligned} \textcircled{1} \quad \frac{x^2 - y^2}{x + y} &= \frac{x^2 - y^2}{x + y} \cdot \frac{1}{x^4 - y^4} \\ &= \frac{\cancel{x^2 - y^2}}{x + y} \cdot \frac{1}{(x^2 + y^2)(\cancel{x^2 - y^2})} \\ &= \frac{1}{(x + y)(x^2 + y^2)} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \frac{3x^2 - 6x}{x^2 - 6x + 9} \cdot \frac{x^2 - x - 6}{x^2 - 4} &= \frac{3x(\cancel{x - 2})}{(\cancel{x - 3})(x - 3)} \cdot \frac{(\cancel{x - 3})(x + 2)}{(x + 2)(\cancel{x - 2})} = \boxed{\frac{3x}{x - 3}} \end{aligned}$$

Part 8

$$\text{LCD: } t(t-1)(t+1)$$

$$\textcircled{1} \quad \frac{1}{t^2+t} + \frac{1}{t^2-t} = \frac{\cancel{t-1} \cdot 1}{\cancel{t-1} t(t+1)} + \frac{1}{t(t-1)} \frac{\cancel{t+1}}{\cancel{t+1}}$$

$$= \frac{t-1 + t+1}{t(t-1)(t+1)}$$

$$= \frac{t}{t(t-1)(t+1)}$$

$$= \boxed{\frac{t}{(t-1)(t+1)}} \text{ OR } \boxed{\frac{t}{t^2-1}}$$

$$\textcircled{2} \quad \frac{x}{x-a} \frac{\cancel{x+a}}{\cancel{x+a}} - \frac{x^2+a^2}{x^2-a^2} + \frac{a}{x+a} \frac{\cancel{x-a}}{\cancel{x-a}} \quad \text{LCD: } (x+a)(x-a)$$

$$\frac{x(x+a) - x^2 - a^2 + a(x-a)}{(x+a)(x-a)}$$

$$\frac{\cancel{x^2} + ax - \cancel{x^2} - a^2 + ax - a^2}{(x+a)(x-a)}$$

$$\frac{-2a^2 + 2ax}{(x+a)(x-a)}$$

$$\rightarrow \frac{2a(x-a)}{(x+a)\cancel{x-a}}$$

$$\frac{2ax - 2a^2}{(x+a)(x-a)}$$

$$\boxed{\frac{2a}{x+a}}$$

Part 9

$$\begin{aligned}
 \textcircled{1} \quad & \frac{\frac{2}{y+2} - \frac{1(y+2)}{1(y+2)}}{\frac{1}{y+2} + \frac{1(y+2)}{1(y+2)}} = \frac{\frac{2-(y+2)}{y+2}}{\frac{1+y+2}{y+2}} = \frac{\frac{2-y-2}{y+2}}{\frac{y+3}{y+2}} = \\
 & = \frac{-y}{y+2} \cdot \frac{y+2}{y+3} \\
 & = \boxed{\frac{-y}{y+3}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad & \frac{\frac{(k-2)k}{(k-2)1} + \frac{1}{k-2}}{\frac{k^2}{k-2} + \frac{1(k-2)}{1(k-2)}} = \frac{\frac{k(k-2)+1}{k-2}}{\frac{k^2+k-2}{k-2}} = \frac{\frac{k^2-2k+1}{k-2}}{\frac{k^2+k-2}{k-2}} = \\
 & = \frac{k^2-2k+1}{k-2} \cdot \frac{k-2}{k^2+k-2} \\
 & = \frac{(k-1)(k-1)}{(k+2)(k-1)} \\
 & = \boxed{\frac{k-1}{k+2}}
 \end{aligned}$$

Part 10

$$\text{LCD: } (z-3)(z+3)$$

$$\textcircled{1} \quad \frac{z+3}{3-z} + \frac{11z+3}{z^2-9} = \frac{1-5z}{z+3}$$

$$(z+3)(z-3) \left(\frac{z+3}{-1(z-3)} + \frac{11z+3}{z^2-9} \right) = \left(\frac{1-5z}{z+3} \right) (z+3)(z-3)$$

$$\frac{(z+3)(z+3)}{-1} + 11z+3 = (1-5z)(z-3)$$

$$\frac{z^2+6z+9}{-1} + 11z+3 = z-3-5z^2+15z$$

$$\begin{array}{r} -z^2-6z-9+11z+3 = 16z-3-5z^2 \\ +5z^2 \qquad \qquad \qquad +5z^2 \end{array}$$

$$\begin{array}{r} 4z^2+5z-6 = 16z-3 \\ -16z \qquad -16z \end{array}$$

$$\begin{array}{r} 4z^2-11z-6 = -3 \\ +3 \end{array}$$

$$4z^2-11z-3=0$$

$$(4z+1)(z-3)=0$$

$$\begin{array}{r} 4z+1=0, \quad z-3=0 \\ -1 \quad -1 \quad +3 \quad +3 \end{array}$$

$$\frac{4z}{4} = \frac{-1}{4} \quad \times \quad \boxed{z=3}$$

CK
does not work ☹
(you should do it)

✓ $\boxed{z=-\frac{1}{4}}$ → CK WORKS! (you should do it)

Part 10 (Continued)

$$\textcircled{2} \frac{(y-3)^2 (y+1)^2}{(y-3)^2} = 1 (y-3)^2$$

$$(y+1)^2 = (y-3)^2$$

$$\begin{array}{r} y^2 + 2y + 1 \\ -y^2 \end{array} = \begin{array}{r} y^2 - 6y + 9 \\ -y^2 \end{array}$$

$$\begin{array}{r} 2y + 1 = -6y + 9 \\ +6y \quad \quad +6y \end{array}$$

$$\begin{array}{r} 8y + 1 = 9 \\ -1 \quad -1 \end{array}$$

$$\begin{array}{r} 8y = 8 \\ \underline{8} \quad \underline{8} \end{array}$$

$$\boxed{y = 1}$$

CK: It works! (But you still need to do it! 😊)

Part 17

$$\textcircled{1} \sqrt{2x-2} - \sqrt{x+6} = 1$$

$$+ \sqrt{x+6} \quad + \sqrt{x+6}$$

$$(\sqrt{2x-2})^2 = (1 + \sqrt{x+6})^2$$

$$2x-2 = 1 + 2\sqrt{x+6} + x+6$$

$$\begin{array}{r} 2x-2 = x+7+2\sqrt{x+6} \\ -x \quad \quad -x \end{array}$$

$$\begin{array}{r} x-2 = 7+2\sqrt{x+6} \\ -7 \quad -7 \end{array}$$

$$\begin{array}{r} x-9 = 2\sqrt{x+6} \\ \hline 2 \end{array}$$

$$\left(\frac{x-9}{2}\right)^2 = (\sqrt{x+6})^2$$

$$4 \left(\frac{x^2 - 18x + 81}{4} \right) = 4(x+6)$$

$$\begin{array}{r} x^2 - 18x + 81 = 4x + 24 \\ -4x \quad -4x \end{array}$$

$$\begin{array}{r} x^2 - 22x + 81 = 24 \\ -24 \end{array}$$

$$x^2 - 22x + 57 = 0$$

$$(x-3)(x-19) = 0$$

$$x-3=0, x-19=0$$

~~$x=3$~~
CK
doesn't
work

$x=19$
CK
works!

→ extraneous
(but you should
do the
CK)

Part 17 Continued

$$\textcircled{2} \quad 3x = \cancel{2} + x\sqrt{5}$$
$$\quad \quad \quad -2 \quad \quad \quad -2$$
$$(3x-2)^2 = (x\sqrt{5})^2$$

$$9x^2 - 12x + 4 = x^2 \cdot 5$$

$$9x^2 - 12x + 4 = \cancel{5x^2}$$
$$\quad \quad \quad -5x^2 \quad \quad \quad -5x^2$$

$$4x^2 - 12x + 4 = 0$$

$$4(x^2 - 3x + 1) = 0$$

doesn't factor; Use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad a=1, b=-3, c=1$$

$$= \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-4}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

$$\frac{3+\sqrt{5}}{2} \quad \quad \quad \cancel{\frac{3-\sqrt{5}}{2}}$$

extraneous

Don't worry
about
this one.

7CK:

$$3\left(\frac{3-\sqrt{5}}{2}\right) \stackrel{?}{=} 2 + \left(\frac{3-\sqrt{5}}{2}\right)\sqrt{5}$$

$$9 - \frac{3\sqrt{5}}{2} \stackrel{?}{=} \frac{2 + 3\sqrt{5} - 5}{2}$$

$$\frac{9 - 3\sqrt{5}}{2} \stackrel{?}{=} \frac{-3 + 3\sqrt{5}}{2}$$

both don't CK.